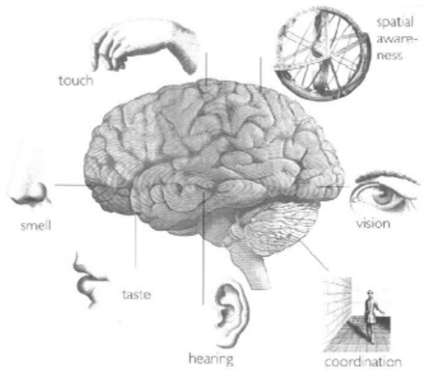
THE NEUROPSYCHOLOGY OF MATHEMATICS



Steven G. Feifer, D.Ed., NCSP, ABSNP
School Psychologist

Presentation Of Goals

(1) Discuss the primary numeric abilities inherent in all species, not just human beings.

(2) Introduce a *brain-based* educational model of math by identifying three basic neural codes which format numbers in the brain.

(3) Explore the role of various cognitive constructs including working memory, visual-spatial functioning, and executive functioning, with respect to math problem solving ability.

(4) Explore the role of anxiety as it relates to gender differences in math aptitude

(5) Introduce the 90 minute assessment model of mathematics and interventions.

TIMSS DATA 4th Grade

Country	Average Score
International Average	*495*
Singapore	594
Hong Kong	575
Japan	565
Chinese Taipei	564
Belgium-Flemish	551
Netherlands	540
Latvia	536
Lithuania	534
Russian Federation	532
England	531
Hungary	529
UNITED STATES	518
Cyprus	510
Republic of Moldova	504
Italy	503
Australia	499
New Zealand	493

TIMSS DATA 8th Grade

Country	Average Score
International Average	*466*
Singapore	605
Korea	589
Hong Kong	586
Chinese Taipei	585
Japan	570
Belgium-Flemish	537
Netherlands	536
Estonia	531
Hungary	529
Malaysia	508
Latvia	508
Russian Federation	508
Slovak Republic	508
Australia	505
UNITED STATES	504
Lithuania	502
Sweden	499
Scotland	498
Israel	496

PISA DATA: 15 yr. olds

Country	Average Score
International Average	500
Finland	544
Korea	542
Netherlands	538
Japan	534
Canada	532
Belgium-Flemish	529
Switzerland	527
Australia	524
New Zealand	523
Czech Republic	516
Iceland	515
Denmark	514
France	511
Sweden	509
Austria	506
Germany	503
Ireland	503
Slovak Republic	498
Norway	495
Luxembourg	493
Poland	490
Hungary	490
Spain	485
UNITED STATES	483
Portugal	466
Italy	466
Greece	445

4 Reasons for U.S Decline

1. The language of math matters! Building number connections centered around a base-10 principle is crucial in the development of mathematical efficiency when problem solving.
2. Dry and boring material. Mathematical skill building needs to be FUN, and therefore needs to be presented in the format of games and activities.
3. Too much focus on the answers. In order to become facilitators of mathematical knowledge, students should practice multiple methods of problem solving from both a visual-spatial and verbal approach.
4. Time on task. Most elementary math instruction occurs in the afternoon, just 45 minutes per day.

4 Common Fallacies Associated with Math

(1) Math abilities are a by-product of IQ.

- **Most animals can subitize, estimate numbers, and comprehend *"more"* and *"less"* comparable to an infant (Lakoff & Nunez, 2000).**
- **Numeric abilities in babies include discriminating up to four objects. One week-old babies are sensitive to numerosity (Antell & Keating, 1983)**
- **In chimpanzees, numeral memory and sense of numerosity equivalent to most preschool children (Kawai & Matsuzawa, 2000).**
- **"*Calendrical" Calculations***

4 Common Fallacies Associated with Math

(2) Math is a right hemispheric task.

- ***"Triple-Code Model* of mathematics suggest that multiple neural networks are involved in the processing of stored quantitative knowledge (Dehaene & Cohen, 1997).**
- **Left hemisphere dominant for most academic tasks including mathematics.**

4 Common Fallacies Associated with Math

(3) Boys outperform girls in math.

- **No evidence at the elementary level, though gender differences emerge in late high school and college (Hyde et al, 1990).**
- **NAEP (2000) revealed gap between boys and girls evident at High School has remained fairly small over the past 10 years.**
- **Males over-represented at both high and low end of the distribution (Casey, et al, 1997).**

4 Common Fallacies Associated with Math

(4) Math is independent of language.

- **Verbal retrieval for archived information is vital to learning over-learned facts such as multiplication tables and basic addition and subtraction facts.**
- **The language of math is critical to comprehending basic word problems (Levine & Reed, 1999).**
- **Math is *interdependent* on language!!**

Geary's 4 Biologically Driven Quantitative Abilities (Pre-verbal)

1. ***Subitizing*** **- the ability to determine the quantity of small sets without counting (max=4).**
2. ***Ordinality*** **- basic understanding of "*more " or "less"* shared by most animals in the wild.**
3. ***Counting*** **- pre-verbal counting system up to 4 objects. Serial counting represents an innate mathematical syntax of numbers.**
4. ***Arithmetic*** **- sensitivity to combining and decreasing quantities in small sets.**

Language systems enhance all of these innate sets of skills

The Neural Machinery of Mathematics

Basic Terminology:

- ***Math Disability (Dyscalculia)*****- refers to children with markedly poor skills at deploying basic computational processes used to solve equations (Haskell, 2000). These may include deficits with:**

  (1) Poor language and verbal retrieval skills
  (2) Working memory skills
  (3) Executive functioning skills
  (4) Faulty visual-spatial skills

The Neural Machinery of Mathematics

Language Skills: (temporal lobes)

- Early math skills tend to be verbally encoded.
- Most Asian language have linguistic counting systems past *ten* (*ten-one, ten-two, etc)* whereas English deviates from base-10 system (Campbell & Xue, 2001)
- Children with math disabilities frequently have delays in their language development. (Shalev et al, 2000)
- Word problems offer an intricate relationship between language and mathematics. Terms such as *all, some, neither, sum, etc.* may be confusing when embedded in the grammatical complexity of word problems (Levine & Reed, 1999).

13

Linguistic Complexities in Word Problems

(1) Direct Statements
(2) Indirect Statements
(3) Inverted Sequences
(4) Inverted Syntax
(5) Too much information
(6) Semantic ambiguity
(7) Important "little" words
(8) Multiple Steps
(9) Implicit Information

14

The Neural Machinery of Mathematics

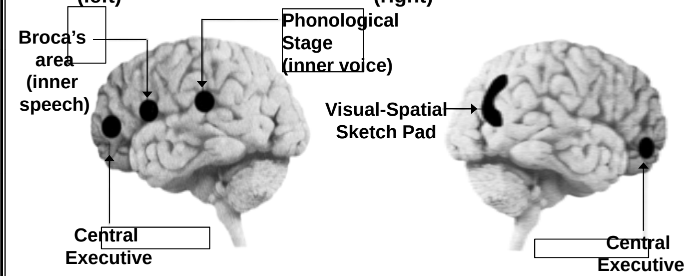
Working Memory Skills: (Baddeley,1998)

- ***Phonological Loop*** - holds and manipulates acoustic information. Housed in *left temporal lobes*.
- ***Visual-Spatial Sketchpad*** - holds visual, spatial, and kinesthetic information in temporary storage by way of mental imagery. Housed along inferior portions of *right parietal lobes.*
- ***Central Executive System*** - command post for controlling two slave systems. Allocates attention resources whereby two cognitive tasks can be executed. *Anterior cingulate* in *frontal lobes*.
  - Central executive system serves to inhibit any negative distractors when problem solving (Hopko, 1998).

15

Working Memory in the Brain

A) Phonological Loop (left) B) Visual-Spatial Sketch Pad (right)



16

Working Memory In The Brain

Working Memory System	Mathematical Skill
• *Phonological Loop*	• *Retrieval of math facts* • *Writing dictated numbers*
• *Visual-Spatial Sketchpad*	• *Mental math* • *Magnitude comparisons* • *Geometric Proofs*
• *Central Executive System*	• *Transcoding mental operations* • *Deciphering word problems* • *Determining plausibility of results*

17

The Neural Machinery of Mathematics

<u>Executive Functioning Skills:</u> (frontal lobes)

- **Executive control mechanisms such as planning, self-monitoring, organizing, and allocating attention resources to effectively execute a goal directed task.**
- **Executive functioning dictates "*what to do when*", a critical process in solving word problems.**
- **Executive functioning allows students to follow an algorithm when problem solving.**

18

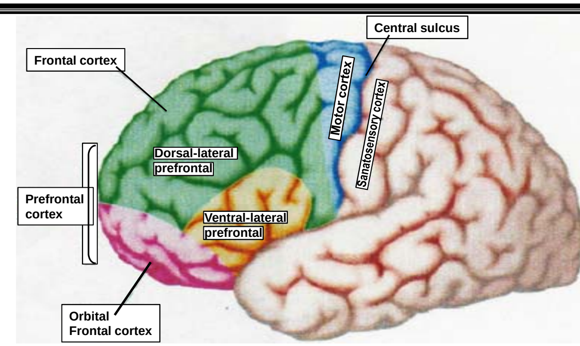
The Neural Machinery of Mathematics

<u>Executive Functioning Skills:</u>

- ***<u>Dorsal-lateral cortex</u>* - helps to organize a behavioral response to solve complex problem solving tasks.**
- ***<u>Orbitofrontal cortex</u>* - rich interconnections with limbic regions and helps modulate affective problem solving, judgement.**
- ***<u>Anterior cingulate cortex</u>* - allocates attention resources and modulates motivation.**

19

The Neural Machinery of Mathematics



- ***Orbital frontal cortex* is end point for ventral stream**
- ***Dorsal-lateral cortex* is end point for dorsal stream**

20

The Neural Machinery of Mathematics

EXECUTIVE DYSFUNCTION	BRAIN REGION	MATH SKILL
• *Selective Attention*	• *Anterior Cingulate*	• Procedure/algorithm knowledge impaired • Poor attention to math operational signs • Place value mis-aligned
• *Planning Skills*	• *Dorsal-lateral PFC*	• Poor estimation • Selection of math process impaired • Difficulty determining salient information in word problems

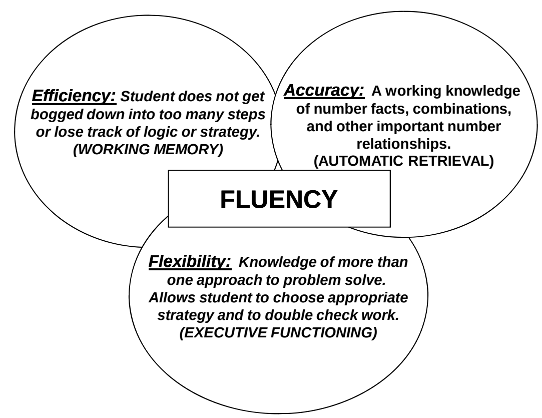
The Neural Machinery of Mathematics

EXECUTIVE DYSFUNCTION	BRAIN REGION	MATH SKILL
• *Organization Skills*	• *Dorsal-lateral PFC*	• Inconsistent lining up math equations • Frequent erasers • Difficulty setting up problems
• *Self-Monitoring*	• *Dorsal-lateral PFC*	• Limited double-checking of work • Unaware of plausibility to a response. • Inability to transcode operations such as (4X9) = (4X10) – 4

The Neural Machinery of Mathematics

EXECUTIVE DYSFUNCTION	BRAIN REGION	MATH SKILL
• *Retrieval Fluency*	• *Orbital frontal PFC* • *Anterior Cingulate* • *Dorsolateral PFC* *(dictated by strategy and effort)*	• Slower retrieval of learned facts • Accuracy of recall of learned facts is inconsistent

MATH FLUENCY (Russell, 1999)



Teacher Concerns With Mathematics

" Johnny does fine on rote calculation items, but cannot solve math word problems."

" Johnny cannot seem to begin an assignment on his own, but once he gets started, he does fine."

" Johnny copies numbers fine from the board, but cannot get anything down on paper from dictation."

" Johnny can add and subtract single digit numbers, but cannot calculate longer math equations."

" Johnny over-relies on manipulatives when doing math."

Error Analysis of Math Facts

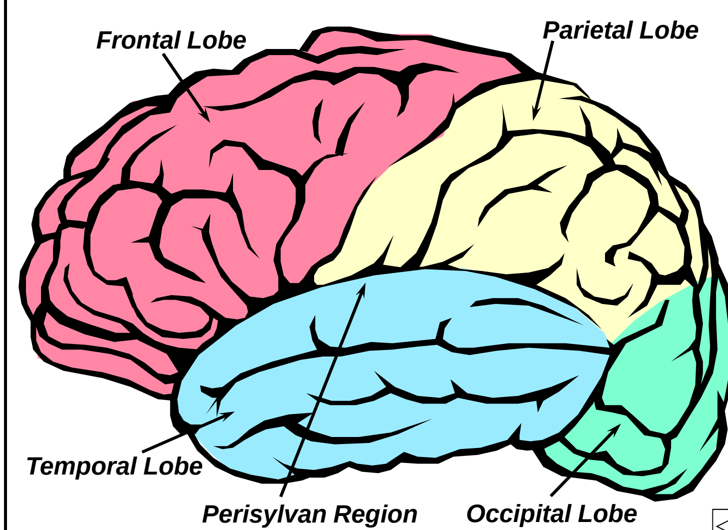
Error Type	Example	Math Subtype
• Math Fact Error:	6+5=10	Verbal Retrieval
• Operand Error:	6-5=11	Procedural error due primarily to poor attn or exec funct.
• *Algorithm Error:	123 -87 44	Procedural due to poor working memory.
• Place Value Error:	.70 +.75 .145	Procedural due to poor working memory.
• Word Problem Difficulties:		Verbal dysfunction

* Semantic deficits may be noted as well

Three Basic Neural Codes to Format Numbers in the Brain

(1) ***Verbal Code*** - numbers are encoded as sequences of words (*twenty-four* instead of 24).
- Dehaene & Cohen, 1997

- Left perisylvan region of temporal lobes.
- No need to understand quantitative concept.
- Main strategy used by younger children learning basic math facts (*two plus two equals four)*
- Critical for memorization of over-learned facts, such as multiplication facts (*nine times nine equals eighty-one)*.

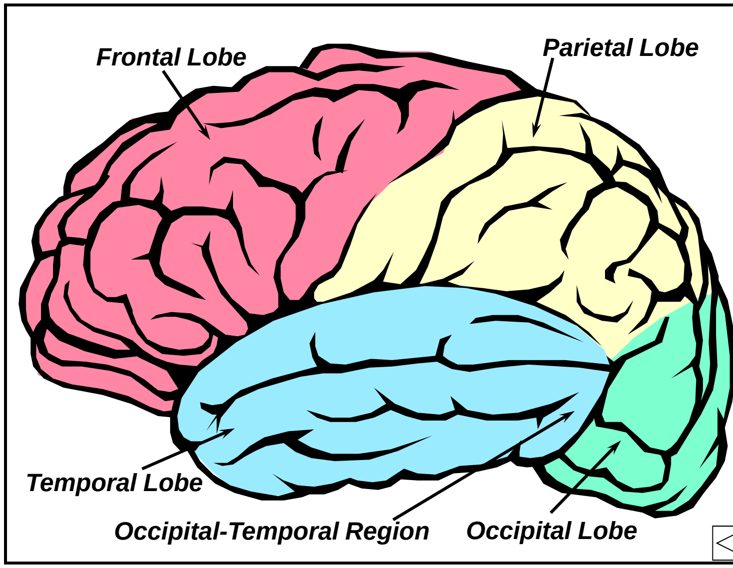


Three Basic Neural Codes to Format Numbers in the Brain

(2) Procedural Code **- numbers are encoded as fixed symbols representing a quantity of some sort, and sequenced in a particular order. (*24 instead of twenty-four*). - Von Aster, 2000**

- **Bi-lateral occipital-temporal lobes.**
- **Critical for number identification skills.**
- **Circuitry involves the syntactical arrangement of numerals. Our internal number line.**
- **Critical in the execution of mathematical procedures for equations not committed to rote memory (e.g. subtraction with regrouping).**

29

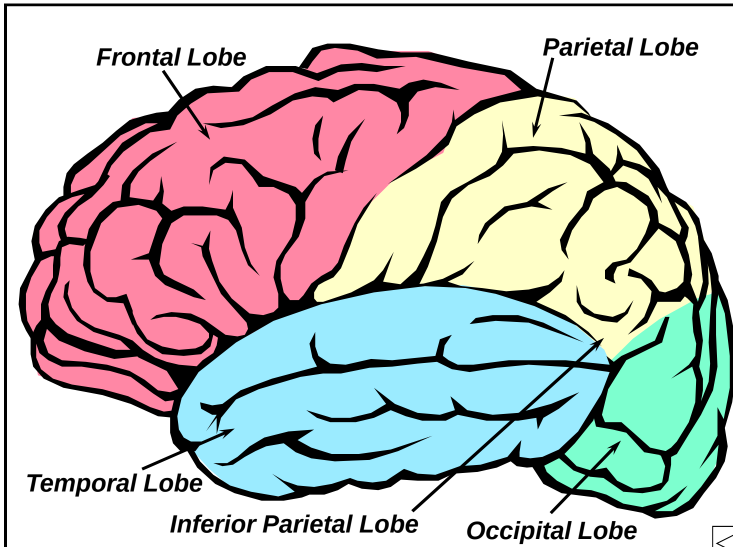


Three Basic Neural Codes to Format Numbers in the Brain

(3) *Magnitude Code* - numbers are encoded as analog quantities. Allows for value judgements, such as *"9"* is bigger than *"4". (*Chocon, et al, 1999)

- **Bi-lateral inferior parietal lobes.**
- **Allows for semantic understanding of math concepts and procedures.**
- **Allows for the evaluation of the plausibility of a response. (9 X 4 = 94)**
- **Allows for the transcoding of more challenging tasks into palatable forms of operations.**
- **(15 percent of 80 becomes 10 percent of 80 plus half the value)**

31



Summary of Triple Code Model

MATH SKILL	BRAIN REGION
Addition Facts	Perisylvan Region Left Hem.
Multiplication Facts	Perisylvan Region Left Hem.
Regrouping Skills	Bi-lateral Occipital-Temporal
Long Division	Bi-lateral Occipital-Temporal
Estimation Skills	Bi-lateral Inferior Parietal Lobe
Geometric Proofs	Bi-lateral Inferior Parietal Lobe
Fractions	Bi-lateral Inferior Parietal Lobe

3 Subtypes of Math Disabilities

(1) *Verbal Dyscalculia Subtype*:
Main deficit is the automatic retrieval of number facts which have been stored in a linguistic code.

- **Multiplication and addition often impaired.**
- **Poor at math fluency tests.**
- **Math algorithms often preserved.**
- **Often have learning disabilities in language arts as well.**

KEY CONSTRUCT: Verbal Retrieval Skills

3 Subtypes of Math Disabilities

(1) *Verbal Dyscalculia Interventions*:
(Wright, Martland, & Stafford, 2000)

- **Distinguish between reciting *number words,* and *counting* (words correspond to number concept).**
- **Develop a FNWS and BNWS to *ten, twenty*, and *thirty* without counting back. Helps develop an automatic retrieval skills.**
- **Develop a base-ten counting strategy whereby the child can perform addition and subtraction tasks involving tens and ones.**
- **Reinforce the language of math by re-teaching quantitative words such as *more, less, equal, sum, altogether, difference, etc...***

3 Subtypes of Math Disabilities

(2) *Procedural Dyscalculia Subtype*:
A breakdown in comprehending the syntax rules in processing and encoding numeric information. Often associated with deficits in working memory.

- **Difficulty writing numbers from dictation.**
- **Subtraction and division often impaired.**
- **Retrieval of math facts and magnitude comparisons often preserved.**

Key Constructs: Working Memory and Anxiety

3 Subtypes of Math Disabilities

(2) *Procedural Dyscalculia Interventions*:

- **Freedom from anxiety in class setting. Allow extra time for assignments and eliminate fluency drills.**
- **Color code math operational signs and pair each with pictorial cue.**
- **Talk aloud all regrouping strategies.**
- **Use graph paper to line up equations.**
- **"*Touch math*" to teach basic facts.**
- **Attach number-line to desk and provide as many manipulatives as possible when problem solving.**
- **Teach skip-counting to learn multiplication facts.**

37

3 Subtypes of Math Disabilities

(3) *Semantic Dyscalculia Subtype*:
A breakdown in comprehending magnitude representations between numbers and understanding the spatial properties of numeric relations. Can be associated with lower IQ and faulty executive functioning skills.

- **Difficulty evaluating the plausibility of a response (e.g. 2 X 4 = 24)**
- **Inability to transcode math operations into a more palatable form (e.g. 9 X 4 is same as (4 X 10) - 4).**
- **Poor magnitude comparisons.**

Key Constructs: IQ, Executive Functioning, Visual-Spatial

38

3 Subtypes of Math Disabilities

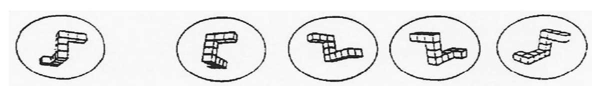
(3) *Semantic Dyscalculia Interventions*:

- **Reinforce basic pattern recognition skills by sorting objects by size and shape.**
- **Have students explain their strategies when problem solving to expand problem solving options.**
- **Teach estimation skills to allow for effective previewing of response.**
- **Have students write a math sentence from a verbal sentence.**
- **Construct incorrect answers to equations and have students discriminate correct vs. incorrect responses.**
- **Incorporate money and measurement strategies to add relevance. Use "*baseball*" examples as well.**

39

The Anxious Brain and Mathematics

- **Math Anxiety + Time Constraints = Poor Performance**
- **Males have stronger SAT math scores than females (SAT mean boys = 595 vs SAT mean girls = 554)**
- **Girls reported more anxiety and less self-confidence on visual spatial problem solving tasks (Casey, 1997)**



Correct answers: The first and second

Figure 1. A sample test item from the Vandenberg Test of Mental Rotation with the standard on the left and the four choices on the right. From *Vandenberg Test of Mental Rotation* by S. G. Vandenberg, 1978, unpublished test. Copyright 1978 by S. G. Vandenberg. Reprinted with permission.

40

The Anxious Brain and Mathematics

SUMMARY: MATH ANXIETY (Casey, 1997):

- **Math anxiety alone not solely responsible for differences between boys and girls.**
- **Students with cognitive flexibility to use either a verbal or a visual-spatial strategy when solving a math problem are inherently less likely to become anxious than students with a singular methodology.**
- **Anxiety itself may serve as a double-edged sword in that the more anxious we become, the less cognitive flexibility we have to use alternative problem solving strategies.**

41

The Anxious Brain and Mathematics

Working Memory and Anxiety:

- **Students with elevated levels of math anxiety perform more poorly than students with lower math anxiety on all levels of mathematical problem solving (Kellogg et al, 1999).**
- **Central executive system, which functions to inhibit negative distracters, is often rendered useless when anxious (Anterior Cingulate). This paves the way for worrisome and negative thoughts which overburden the system (Hopko et al, 1998).**

42

5 Ways to Reduce Math Anxiety

(1) Teach multiple ways to problem solving :

- **Students who utilize both visual/spatial and verbal strategies outperform students who over-rely on just a singular methodology**

(2) Avoid skill drills:

- **Speed and competition creates anxiety. Obtain fluency without classroom competitions.**

(3) Link problem solving with passion:

- **Attach personal meaning to the harshness of cold problem solving. Baseball batting averages or Yu-Gi-Oh life points are a fun way to learn math operations.**

43

5 Ways to Reduce Math Anxiety

(4) Set algorithmic procedures to a song:

- **Lower anxiety leads to better working memory and better working memory lowers anxiety. Set math operational steps to a song or rap, as verbal strategies often serve as a memory enhancer.**

(5) Encourage visual cues:

- **Students who take the time and effort to jot down equations on paper as opposed to working out equations in their head, put less stress on working memory systems.**

44

The 90 Minute Mathematics' Assessment

- **Intelligence Tests**
- **Visual-Spatial Functioning**
- **Working Memory Capacity**
- **Executive Functioning Skills**
- **Math Skills and Number Sense**
- **Math Anxiety Scale**
- **Developmental and School History**

45

Assessment Algorithm for Math

(1) Intelligence Tests

WISC IV Construct	Math Implication
Low Verbal Comprehension	* Difficulty with word problems * Poor retrieval of facts * Difficulty with math terms
Low Perceptual Reasoning	* Confusion lining up equations * Poor *mental math* skills * Difficulty with estimation
Low Working Memory	* Forget math steps * Poor regrouping skills * Difficulty with mental rotation
Low Processing Speed	* Difficulty with skill drills * Slower visual pattern rec skills

46

Interventions for Lower Cognitive Skills

- **Manipulatives and hands-on type of instruction.**
- **Number-line situated on student's desk.**
- **Drill and repetition.**
- **Focus on algorithm.**
- **Skip counting.**
- **Tap a drum beat when counting.**
- **Check for plausibility of response.**
- **Have student tell a number story to insure comprehension.**
- **Teach *"math vocabulary"***
- **Utilize music, especially rap, to over-learn facts.**
- **Incorporate an area of passion in all lessons (e.g. baseball statistics, Yu-Gi-Oh life points, NASCAR standings, etc.)**

47

Assessment Algorithm for Math

(2) Visual-spatial Functioning

- **Key Measures From IQ Subtests:**

TEST	SUBTEST
WISC IV	Block Design
WISC IV	Matrices
DAS	Matrices
DAS	Recall of Designs
DAS	Pattern Construction
SBV	Visual-Spatial Processing
SBV	Quantitative Reasoning
WJIII	Spatial Relations
WJIII	Visual Closure
WJIII	Block Rotation

48

Interventions for Poor Visual-Spatial Skills

- Turn a visual problem into a verbal problem.
- Have students *talk* their way through a problem.
- Use graph paper to help line up equations.
- Make sure problems are written vertically as opposed to horizontally.
- Attach number-line to desk.
- Greater emphasis teaching estimation skills and magnitude representations.

Assessment Algorithm for Math

(3) Working Memory Skills

- WISC IV (Digit Span, Letter-Number Sequencing)
- SB5 (Verbal & Nonverbal Working Memory)
- Test of Memory and Learning (Digits & Letters Backwards)
- Trail making Test (Halstead-Reitan)
- Cognitive Assessment System (Planned Connections)
- Children's Memory Scale (Dot Locations, Sequences)
- Woodcock Johnson III (Auditory Working Memory, Numbers Reversed)
- WISC PI (Spatial span, Arithmetic & Sentence Arrangement)
- Wechsler Memory Scale (Visual Reproduction & Paired Associate)
- Paced Auditory Serial Addition Test (PASAT)
- Wide Range Assessment of Memory and Learning – 2nd Ed. (Verbal Working Memory & Symbolic Working Memory)

Interventions for Lower Working Memory

- Number-line situated on student's desk.
- Use a calculator.
- Reduce anxiety in the classroom.
- Increase number sense through games such as dice, domino's, cards, etc..
- Encourage paper and pencil use while calculating equations.
- Use mnemonic techniques to teach math algorithm's and sequential steps to problem solving.

Assessment Algorithm for Math

(4) Executive Functioning Skills

- Executive Functioning Measures:
  - Wisconsin Card Sort Test
  - Stroop Test
  - Category Test
  - Delis-Kaplan Executive Functioning Scale
  - BRIEF
  - NEPSY (Tower)
  - CANTAB (ID-ED Shift)

Assessment Algorithm for Math

(5) Mathematic Skills and Number Sense

- Wechsler Individual Achievement Test- 2nd Edition
- Woodcock Johnson III Achievement Test
- Woodcock Johnson III Cognitive (Number Series & Matrices)
- Test of Early Mathematics Ability – 3rd Edition
- WRAT-3
- NUCALC
- KEYMATH

53

Assessment Algorithm for Math

(5) Mathematic Skills and Number Sense

- NUCALC:
  - Counting dots
  - Counting backwards
  - Dictation of numbers
  - Mental calculations
  - Reading numbers
  - Positioning numbers on analog scale
  - Oral comparison
  - Perceptive estimation
  - Contextual estimation
  - Problem solving
  - Written comparison

54

Assessment Algorithm for Math

(6) Math Anxiety Scales

- Math Anxiety Rating Scale (98 items)
- Abbreviated Math Anxiety Rating Scale (9 items)
- State-Trait Anxiety Inventory
- Behavior Assessment System for Children (BASC)
- Achenbach Child Behavior Checklist
- Piers-Harris Children's Self Concept Scale
- Devereux Scales of Mental Disorders
- Personality Inventory for Children-Second Edition

55

Assessment Algorithm for Math

(6) Math Anxiety Scales

- Abbreviated Math Anxiety Rating Scale (9 items)
  1) Use tables in back of book.
  2) Thinking about test the day before.
  3) Watching teacher work problem on the board.
  4) Taking a math exam.
  5) Math homework due the next day.
  6) Listening to a math lecture.
  7) Listening to another student explain a math formula.
  8) Pop quizzes.
  9) Starting a new math lesson.

56

THREE FINAL THOUGHTS!

1. ***"I never did well in math because I was unable to persuade my teacher not to take my answers literally."***
2. ***"If two wrongs don't make a right, try three!"***
3. ***"If you think dogs can't count, try putting three dog biscuits in your pocket and then give Fido only two of them."***

57